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ALL the articles presented in this *Bulletin* with the exception of the first and the one by James M. Turner, were written by Columbus teachers. These teachers took a "minor research" course at Ohio State University under the direction of Dr. P. R. Stevenson. In this course one phase of the teaching of arithmetic was studied, namely, the difficulties which pupils have in solving verbal problems. A part of the method of procedure in the course was the assignment of topics to committees who investigated the topics and as a result of their investigations and trial of their findings in the classroom prepared the reports submitted herewith.

These reports are now published in the hope that the experience of these teachers will be of benefit to teachers and their supervisors who are interested in improving the instruction in arithmetic.

B. R. BUCKINGHAM, *Editor*

Educational Research Bulletin

Increasing the Ability of Pupils to Solve Arithmetic Problems

REPORT ON PROJECT NO. 40

P. R. STEVENSON
Bureau of Educational Research, Ohio State University

After reading the description of Project 40, Remedial Instruction in Arithmetic,[1] the superintendents of Bridgeton, New Jersey; Crawfordsville, Indiana; Muskogee, Oklahoma; Columbus, Geneva, Youngstown Warren, and Portage County, Ohio, asked to cooperate in the study. This report is based upon data received from 1,014 pupils.

The purpose of the study was to determine the difficulties which pupils had in solving problems and to devise remedial instruction suited to individual cases.

Method of procedure.—In order to find individual disabilities and to measure the results of remedial instruction, tests were given at the beginning of the experiment and again at the end of the period of remedial instruction. The tests used were the Buckingham Scale for Problems in Arithmetic and the Stevenson Problem Analysis Test.[2]

REMEDIAL INSTRUCTION

The remedial instruction was planned to cover a period of twelve weeks. All the work was done in the regular recitation period. Three fifteen-minute periods each week were devoted to special remedial work making a total of nine hours.

First, second, and third weeks.—During these three weeks the pupils were taught to read and analyze problems. They were directed to open their books to a page containing a list of problems. Before solving a problem they were drilled in finding what facts were given in the problem, what question was asked, the process or different processes which should be used in solving the problem, and the answer in round numbers. A list of problems was also written on the board and pupils instructed to answer the above types of questions either orally or in writing. The teacher studied the results of the analysis test, and saw that each pupil got practice in the exercises upon which he failed. For example, if estimating answers was difficult for John, he was given exercises in this phase of problem drill. Pupils having the same difficulties were given instruction in groups.

Fourth, fifth, and sixth weeks.—During these weeks the pupils worked a large variety of problems containing data from actual life situations. The pupils submitted lists of problems which they and their parents had occasion to solve. The teachers tried to

(1) This project was described in detail in the January 9, 1924, issue of the *Educational Research Bulletin.*

(2) Both tests are published by the Public School Publishing Company, Bloomington, Illinois.

arouse the pupils' interest in solving many different types of problems.

Seventh, eighth, and ninth weeks.—During this time the pupils solved problems without the use of numbers. Such problems were made up by the teachers and adapted as far as possible to each grade. The following is an illustration of the type of problems used:

If you knew the amount of cloth needed to make a dress, how could you find the amount necessary for three dresses?

Tenth, eleventh, and twelfth weeks.—During these weeks pupils studied the difficult words in problems. The following devices were used: (1) Pupils stated problems in their own words and each problem was given in as many different ways as possible. (2) In various problems words were studied which might cause difficulty. The children explained the meanings of such words. A list of words for study in the different grades was sent to teachers who cooperated.

TABLE I. CLASS RECORD SHEET. PROJECT 40

NAME	BUCKINGHAM PROBLEM SCALE			STEVENSON PROBLEM ANALYSIS TEST														TOTAL		
				A			B			C			D							
	Trial		Gain	Trial		Gain	Trial		Gain	Trial		Gain	Trial		Gain	Trial		Gain	Trial		Gain
	1	2		1	2		1	2		1	2		1	2		1	2		1	2	
1. Philip	63	70	7	6	5	−1	6	6	.....	6	6	.....	6	6	.....	24	23	−1			
2. Rex	63	60	−3	2	4	2	6	6	.....	3	4	1	3	6	3	14	20	6			
3. Nellie	65	64	−1	6	6	.....	6	6	.....	5	6	1	6	6	.....	23	24	1			
4. Cleatus	61	66	5	2	6	4	6	6	.....	6	6	.....	6	6	.....	20	24	4			
5. Dorothy	61	70	9	5	6	1	4	6	2	4	5	1	5	6	1	18	23	5			
6. Mildred	61	62	1	5	6	1	6	6	.....	5	6	1	6	6	.....	22	24	2			
7. Alfred	59	62	3	4	6	2	6	6	.....	5	6	1	5	6	1	20	24	4			
8. Helen	59	68	9	5	6	1	5	6	1	4	5	1	5	6	1	19	23	4			
9. Fred	59	64	5	5	6	1	4	6	2	5	5	.....	5	6	1	19	23	4			
10. Herbert	59	68	9	5	5	.....	5	5	.....	4	6	2	4	6	2	18	22	4			
11. Ruby	59	64	5	3	5	2	5	6	1	3	5	2	4	6	2	15	22	7			
12. Norma	57	62	5	5	6	1	6	6	.....	3	6	3	6	6	.....	20	24	4			
13. Fred	57	60	3	5	6	1	6	6	.....	3	5	2	5	6	1	19	23	4			
14. Jack	57	64	7	4	6	2	5	5	.....	5	6	1	6	6	.....	20	23	3			
15. Elizabeth	53	60	7	3	5	2	5	6	1	4	6	2	5	6	1	17	23	6			
16. Lewis	51	62	11	5	4	−1	6	6	.....	4	4	.....	6	6	.....	21	20	−1			
17. Elizabeth	51	60	9	5	6	1	5	6	1	3	5	2	4	6	2	17	23	6			
18. Mildred	51	46	−5	4	4	.....	4	6	2	3	4	1	3	6	3	14	20	6			
19. Lois	49	62	13	4	6	2	5	6	1	3	5	2	3	6	3	15	23	8			
20. Charles	49	54	5	4	6	−2	2	3	1	2	4	2	3	6	3	11	19	8			
21. Janet	47	52	5	3	2	−1	5	4	−1	2	5	3	2	6	4	12	17	5			
22. Billy	47	64	17	5	5	.....	5	6	1	2	6	4	5	6	1	17	23	6			
23. Harold	44	64	20	4	6	2	4	6	2	2	3	1	4	6	2	14	21	7			
24. Erwin	42	60	18	3	6	3	5	6	1	3	6	3	3	6	3	14	24	10			
25. Dorothy	42	46	4	3	5	2	5	6	1	2	5	3	2	3	1	12	19	7			
26. Ruby	42	54	12	2	3	1	5	4	−1	2	4	2	3	5	2	12	16	4			
Total	1408	1588	180	107	137	30	132	147	15	93	134	41	115	152	37	447	570	123			
Average	54.15	61.08	6.92	4.12	5.27	1.15	5.08	5.65	.58	3.58	5.15	1.58	4.42	5.85	1.42	17.19	21.92	4.73			

RESULTS

The record sheet shown in Table I was used for recording pupils' scores. On the problem test, Philip, pupil Number 1, made 63 for the first trial and after the period of remedial instruction he raised his score to 70. He also made a slight gain on the problem-analysis test. The latter test, however, was too easy for him, since he obtained almost a perfect score on the first trial. Similar details are given for each pupil, showing also which phase of problem solving caused difficulty. The average class score is presented in the last line. Since the Buckingham Standard for grade VI-low is 65 and for grade V-LOW is 56, it is evident that this class, with an average score of 54 on the first trial, was doing inferior work in problem solving. Even after a period of special instruction they succeeded in scoring only 61, the standard for grade V-high.

Table II shows the average class scores for the sixth grade. The figures for class Number 12 are given in Table I. It will be seen that all classes except Number 25 fall below the standard for grade VI-low in the first trial on the problem test. Each class, however, made considerable improvement in the second trial. The average gain on the problem test made by the sixth-grade classes (64.4—54.3) was 10.1. Since, according to the norms, a gain of 5 points is equivalent to the growth made in a semester, it is evident that these sixth-grade classes made a year's improvement as the result of twelve-weeks remedial instruction.

In respect to pupils' abilities to read and analyze problems, it will be noted from the last line and the last two columns that the first score was 16.5 and the second score, 20.5. This gain of 4 represents a growth of approximately two years according to tentative norms on the analysis test. It must be borne in mind, however, that the pupils also acquired a method of solving and analyzing problems independent of specific teaching.

Average scores for grades V, VI, and VII are given in Table III. The results for grade VI are those which are shown in the last line of Table II.

TABLE II. AVERAGE SCORES* BEFORE AND AFTER REMEDIAL INSTRUCTION FOR GRADE VI

CLASS NUMBER	NUMBER OF PUPILS	PROBLEM TEST AVERAGE SCORES*		PROBLEM ANALYSIS AVERAGE SCORES*	
		Trial		Trial	
		1	2	1	2
1.....	33	57.6	68.	18.	21.9
2.....	23	54.3	65.2	16.7	22.2
3.....	25	56.8	63	18.3	21.1
4.....	27	55.4	61.9	16.	20.8
5.....	35	50.3	55.5	15.9	20.2
6.....	31	56.4	65.5	18.	21.8
7.....	17	57.7	70.0	16.9	23.4
8.....	27	54.4	64.3	19.1	21.7
9.....	26	56.8	67.2	17.7	22.0
10.....	17	54.2	67.3	15.6	21.9
11.....	16	56.9	70.1	18.1	22.6
12.....	26	54.2	61.1	17.2	21.9
13.....	34	56.1	63.8	19.1	22.4
14.....	16	56.4	64.1	16.6	22.5
15.....	16	58.3	66.6	15.5	21.7
16.....	16	56.4	65.4	16.2	21.4
17.....	28	59.	67.	17.8	22.4
18.....	17	17.	49.	1.	14.
19.....	32	60.2	66.3	17.	21.2
20.....	25	54.	66.	16.	20.
21.....	34	57.	65.	16.4	19.6
22.....	30	43.4	66.9	13.5	17.6
23.....	33	48.2	59.3	16.8	18.0
24.....	49	53.0	63.0	15.0	16.0
25.....	18	72.1	77.1	18.4	20.8
Total.....	651	35329.7	41926.6	10709.5	13365.8
Average.....		54.3	64.4	16.5	20.5

* All averages are weighted averages.

An examination of the scores from first and second trials on the problem test will show that grade VI profited most by the instruction; grade V, a little less; and grade VII, least of all. The average gain on the problem test of 8.6 (62.6—54.8=8.8) for all grades equals a little more than four-fifths of a year.

The question naturally arises as to whether bright, average, or dull pupils benefited most by the remedial instruction. Gross gains are not accurate measures of pupil improvement since it is easier for a pupil to make a gross gain of ten from a zero score than from a score of fifty or ninety. To determine therefore, the relative improvement of different pupils, all gross scores were converted into T-Scores.[3]

TABLE III. AVERAGE SCORES FOR GRADES V, VI AND VII BEFORE AND AFTER REMEDIAL INSTRUCTION

GRADE	NUMBER OF CLASSES	NO. OF PUPILS	AVERAGE SCORES*			
			PROBLEM TEST		PROBLEM ANALYSIS	
			Trial		Trial	
			1	2	1	2
V.....	8	265	49.6	55.7	13.2	16.4
VI.....	25	651	54.3	64.4	16.5	20.5
VII.....	5	111	62.6	68.1	14.5	19.3
V, VI, & VII.....	38	1027	54.8	62.6	15.4	19.3

* All averages are weighted averages.

Intelligence tests were given to the sixth-grade pupils in Columbus. For purposes of tabulation the pupils were arbitrarily divided into three groups according to their intelligence quotients. Those having I.Q.'s of 110 upward were considered bright; those having I.Q.'s of 90 to 109.9, as average; and those having intelligence quotients of less than 90, as dull. The average T-Score gains on the problem test made by the different groups are as follows: bright, 2.63; the average, 6.9; and the dull, 8.75. On the problem-analysis test the T-Score gains were, 3.41 for the bright, 5.97 for the average, and 7.29 for the dull. It seems, therefore, that the type of remedial instruction outlined at the beginning of this article and explained in detail by the other writers of this *Bulletin* is least fitted to the bright pupils, while the dull pupils receive the most benefit with the average group as a close second.

In this study no attempt was made to determine the relative value of different methods of teaching problems. It has shown that pupils' ability to solve problems can be increased materially by consciously following several different methods of instruction. More detailed research should be undertaken to show how much time and effort, if any, should be expended in giving pupils a method of attacking problems, in training them to estimate answers, in assisting them to understand technical words, and other phases of instruction discussed in this *Bulletin*.

(3) The space allotted to this article does not permit of an explanation of the T-Score and its significance. The reader may become familiar with the technique by examining the author's monograph, *Larger or Smaller Classes*, published by the Public School Publishing Company, Bloomington, Ill.

Specific Causes of Failure in Arithmetic Problems

PEARL ROLING,
CLARA L. BLUME,
AND
MARY S. MOREHART

The following report outlines briefly some of the important causes for failure in arithmetic problems.

I. *Physical and mental defects.*—Physical defects and lack of mental ability are contributing causes to failure not only in arithmetic but also in other subjects.

II. *Inability to read.*—Pupils often fail to get the meaning of a problem because of word difficulty. They place their attention on word recognition rather than on content. Other pupils neglect to read the problem thoughtfully because by nature they have a motor anxiety to be doing something so they begin making figures. Still other pupils fail because of inability to interpret the meaning of the words used. Such words as *average*, *area*, *rate*, *ratio*, and *product* were some of the words for which we found very indefinite meaning in the minds of pupils.

III. *Lack of variety of good problems.*—The pupils frequently have difficulties because of poorly selected problems. The teachers have long been guilty of trying to adapt pupils to the problems rather than the problems to the pupils. Some of the causes for the lack of variety are:

1. Too few real situation problems are given.
2. Local problems are neglected, if the problem fails to come within the range of the child's own experience, or of what he sees or can readily understand, we cannot expect a great amount of self-activity.
3. Many problems are very indefinite. The author assumes facts not known to the pupils.
4. Some problems are void of all interesting detail and only state the mere facts.
5. Problems with irrelevant elements are confusing to pupils if they have had no experience in dealing with such.
6. Too few problems without numbers are given.
7. Often teachers fail to state problems in varied ways, thus pupils are at sea when the wording is changed.
8. Many problems do not contain conditions that are true.

IV. *Lack of a method for attacking.*—Pupils fail to solve problems, because they lack the power to see relations. They do not use the main elements in problem solving which are: (1) to know just what the question is, (2) to know what facts are to be used to answer the question, and (3) to use them in the right relations.

A frequent cause of mistakes is the inaccurate copying of numbers. This is particularly true of pupils who are naturally left-handed but have been forced to write with their right hands. Many pupils find it difficult to apply information which they have already gained in new problems. Others often fail, because they are limited to a certain type form of analysis and labeling.

V. *Lack of ability in fundamentals.*—Errors in computation are made because the fundamental processes have not been mastered. The solution of the problem cannot be the sole interest of the pupil, if he has to attend to the performance of the operation.

VI. *Poor teaching.*—Teachers do not make proper assignments. They fail to create for the children experiences which they must have in order to solve problems and they also neglect to help the pupils translate problems into their own experiences.

Much time is lost by not teaching pupils to make use of short methods in solving problems. For example, in the problem:

If 3 oranges cost 20 cents, what will be the cost of 12?

It would be absurd to find the cost of one orange and then of twelve when the answer may be derived much more simply by multiplying 4 times 20. A similar oversight is the practice of requiring pupils to put on paper work which could be easily done without a pencil, while the use of estimation and approximation has not been emphasized enough.

Teachers have overlooked the importance of requiring pupils to make their own problems or those for the class. They have spent much time in measuring the results rather than in teaching pupils to make practical application of their knowledge of numbers. Insufficient supplementary work has been given. Many problems of less difficulty are of more value than a few problems of greater difficulty.

Individual difficulties have not been discovered and properly diagnosed. The fact that there are so many causes of failure in arithmetic problems emphasizes the importance of deliberate study of individual difficulties before remedial instruction is applied.

Why Pupils Fail in Fundamentals

MARY F. BROWN,
CLAIRE ALLISON,
ELSA J. FUGE,
AND
LUCY B. SMITH

Notwithstanding the fact that the greater part of the first four years of a child's school-life is spent in mastering the fundamentals of addition, subtraction, multiplication, and division, the percent of failures is appalling. It is the common knowledge of every teacher of arithmetic that the most difficult part of the subject is to secure satisfactory results in fundamentals. A child may correct one mistake in spelling without the need of mastering some previous word or one language error without the knowledge of the correcting principle, but he cannot acquire efficiency in the use of fractions or decimals unless he has acquired skill in the basic processes.

The question, Why do pupils fail in the fundamentals naturally arises? and the answers may generally be found in one or more of several contributing causes:

I. *Lack of necessary drill.* — The requisite preliminary for successful computing is thorough drill on the addition combinations to ten, omitting none, but stressing or varying the amount of drill on the more difficult ones. After a complete mastery of these simpler combinations it will not be a difficult task to teach the remaining of the one hundred. The more thoroughly automatic and mechanical they are made the better will the calculation phases of the problems be performed.

While the purpose of the drill is to render computation quick, accurate, and automatic through the function of the proper habits, accuracy and speed are the aims. Accuracy in the fundamentals is not achieved until necessity for thought in the reckoning has been reduced to a minimum; for it is when we stop to think that mistakes are

made in column addition and other mechanical work. Drill can be depended upon to increase both accuracy and speed—growth in accuracy does not always make equal progress with speed. The former is of more importance than the latter, however, since only results 100 percent correct—the standard of the adding machine—are worth anything. Speed should be acquired through repeated drill and practice, rather than by short methods.

To succeed in our aim we must consider drill and all its concomitants, such as misplaced drill, inadequate drill, and drill to correct wrong fixations, *e. g.*, John learns that 7 times 8 is 54 and consequently all of his examples involving the use of this combination are incorrect. The reason for failures is most often attributed to inadequate drill, but the fault lies in the fact that the drill has not been adapted to the needs of the child. Several children in a class may be having difficulty in the use of zero in multiplication. A teacher may drill tirelessly for many days on the tables, and then discover that she does not seem to have measurably succeeded. If instead she had given several tests involving all the multiplication combinations, as well as the combinations of the other processes, and carefully tabulated each child's errors on an individual score card, she would be able to diagnose each pupil's difficulties. Effective as well as scientific drill could be given to the individual or to small groups.

The individual difficulty card of nine-year-old Thomas showed some interesting errors. In addition he has mistakes in all combinations involving nine. His subtraction tests show that when he subtracts a 0 from any number he has 0 for his answer. In multiplication, like many other children, he insists that 6×0 equals 6, or 0×7 equals 7. His glaring error in division is $9\div9$ equals 9; $8\div8$ equals 8. Patient consistent drill eradicated these mistakes.

II. *Carelessness.*—Next in importance on our list of reasons for failures is carelessness. This includes carelessness not only on the part of the pupils but on the part of the teacher as well. Many teachers disregard the value of having pupils check their work and others permit them to prepare written work which is immediately disposed of without grading. This carelessness and indifference on the part of teachers is soon reflected in the spirit of the class. Does the teacher always exercise the greatest care in placing her work before the class? Are the pupils in danger of confusing a *7* with a *1* or a *0* with a *6*? Does she refuse to accept any slovenly work from her pupils? Does she sin by insisting upon the pupil working too fast or does she err on the side of giving too much time and thereby destroying concentration? These questions should be frankly answered by each instructor before she condemns her class as careless.

On the other hand carelessness on the part of the child will manifest itself in various forms. For instance, the negligent child does not copy the figures of his examples correctly. His discriminating powers in reading signs are poor, *e. g.*, one pupil will add when the multiplication sign is used and divide when the minus sign is given or vice versa. Frequently another pupil arranges his figures in column addition so heedlessly that units are added with the column of tens, or hundreds with the thousands; when he multiplies by a number like 58 he may multiply by the 8 but forget to do the same with the 5. In subtraction, when one is taken and you must still "borrow" from the next higher order very frequently Mary will forget that she must "borrow" again.

III. *Physical drawbacks.*—Following close upon the heels of carelessness and sometimes falsely attributed to it are the errors which are due to physical disabilities and which are sometimes hard to discern. A defect in the vision

or in the auditory nerves may be responsible for errors. Types of amnesia, malnutrition, etc., have also been the cause of mistakes which can be analyzed only by a thorough physical examination of individual cases.

IV. *Defects in the course of study and textbooks.*—A well-organized, complete, and up-to-date course of study in arithmetic should be in every teacher's possession. She should be familiar with the work of the grades preceding and following her own. In this way she will be enabled to carry on successful drill lessons and emphasize the important facts already learned.

Lack of a properly organized course of study leads to a multiplicity of methods of teaching. The most common instance of the confusion caused by this is to be seen in the methods employed to teach subtraction. There are several interpretations of the subtraction concept, "the taking away from" idea, the adding idea, and the difference idea. Many primary teachers conscientiously insist that the additive or Austrian method is the correct method to use; others find the "taking from" plan superior. It must be borne in mind, however, that to successfully teach subtraction the child must be given the threefold concept. A well-planned course of study should incorporate within itself provision for uniform teaching methods, whereas multiplicity of methods leads to confusion.

After a painstaking investigation of methods of this phase of arithmetic it is apparent that these difficulties can be eliminated by a careful study of these obstacles in the teaching of the fundamentals in arithmetic.

How Pupils Solve Problems in Arithmetic

LENORE E. RIMER,
ANNA WILSON,
AND
H. B. KNIGHT

The ability to perform the fundamental processes in arithmetic does not insure ability to solve problems. The severest test comes in dealing with problems in arithmetic, for unless the processes required are glaringly conspicuous the efforts of most pupils are largely guess work, which yields ridiculous results.

A close study of how pupils attack arithmetic problems shows two general types of approach. On the part of the dull pupils we find a mechanical, thoughtless procedure, while the bright pupils make a quick plunge into the middle of the problem, often skipping the opening step or two. Sometimes the result is found mentally, but often the pupil is unable to set his work down on paper.

In order to determine how pupils work, sixteen problems of increasing difficulty were prepared. Bright and dull pupils were then asked to solve them. The observer worked with each child separately. The pupils were asked to think aloud and to tell the observer just what they were doing as they set down their work on the paper. The observer jotted down everything that was said, and in some cases had a stenographer make a complete report. There was no attempt at teaching, since the object was rather to note closely just how pupils attack problems. It was found that five outstanding habits or methods prevailed among the children here considered. The following solutions illustrate these five general classes:

I. The form of the problem

A. Joseph rode on the merry-go-round 12 times. Each time it cost him 3 cents. How much did he pay for all the rides?

Case 1. Girl, 10 years of age, grade IV-B

P.[1] 36
O.[1] How did you get 36?
P. I multiplied.
O. Why did you multiply?
P. Because it said "times."

Case 2. Girl, 11 years old, grade IV-B.

P. 15 cents.
O. Did he? You write that down. You'd better read it over once more to be sure that you have it right. Now, can you do it?
P. Multiply. 12×3=36.
O. Why did you multiply?
P. There *is* only two numbers.
O. Do you know any other reason why you multiply?
P. There *is* just two numbers.

B. If an aeroplane flies 696 miles in 8 hours. how many miles will it fly in one hour?

Case 1. Girl, 11 years of age, grade IV-B.

P. Multiply.
O. Why?
P. 696×8.
O. Why did you multiply?
P. 8 hours are there.
O. Why didn't you add the 8?
P. Wasn't supposed to.
O. How do you know?
P. There wasn't enough numbers to add.
O. What do you mean?
P. There wasn't enough numbers with the 8.
O. Why don't you subtract?
P. I wasn't supposed to. You would have to have other numbers.
O. Why don't you divide? Should you divide or multiply?
P. Divide. It asked how many miles it can fly in one hour.
O. Why did you divide?
P. To see how many miles it would fly in an hour.

Many children determine what process to use by the size of the numbers. They seldom have to subtract or add when they have 696 and 8. They seem to be left to two devices, either to multiply or divide.

(1) Throughout the cases reported in this article *P* will stand for *Pupil; O*, for *Observer*.

II. Many pupils try to do something with all the numbers.

A. The distance from Cincinnati to Cleveland, by way of Columbus, is 262 miles. A train leaving Cleveland at 4:50 will be in Columbus at 7:30 and in Cincinnati at 11:00. If it is 140 miles from Cleveland to Columbus, how far is it from Cincinnati to Columbus?

Case 1. Boy, 12 years old, grade V-A.

P. I am going to add. (Adds all numbers in the problem).
O. Why did you add?
P. You couldn't do anything else because there are so many numbers.

Case 2. Twelve-year-old girl, grade V-B, I. Q. 67.

P. It don't say how far it is from Cincinnati to Columbus?
O. That is what it asks you to find.
P. Yes, but I don't know how far Cincinnati is from Columbus.
O. How are you going to find out?
P. Subtract.
O. What?
P. 140, but I don't know from what.
O. Read it again.
P. I guess you have to add. I can't multiply or divide or subtract. It has to be addition. I'll try it and see. (Puts down all the numbers). You can't add time and miles, so you add time and subtract what you get from 262, Cincinnati must be 211 miles from Columbus.
262+450+750+111=1,473.
1,473—262=211 answer.

B. The pupils in the Irving School agreed to make 675 badges for the school field meet. The day before the meet the third grade turned in 20; the fourth, 40; the fifth, 70; the sixth, 100; the seventh, 200; and the eighth, 200. How many badges were turned in?

Case 1. Boy, 13 years old, Grade V-A.

P. 675 + 20 + 40 + 70 + 100 + 200 + 200 = 1,305.
They turned in 1,305 badges.
O. Why did you add?
P. Because that is the only way I can figure out it would be the best way. All these grades you have to add.
O. Why did you add 675?
P. I don't need that.
O. You don't need it?

P. No, it says the school only needed 675, so I just add the rest. They turned in 630 badges. $20 + 40 + 70 + 100 + 200 = 630$.

III. When pupils do not know the fundamentals, problems are attacked in peculiar ways.

A. Dick made 50 toy boats. He sold 20 of them at 30 cents each, 10 at 25 cents each, 6 at 20 cents, and the rest at 15 cents each. How much did he receive for those he sold at 30 cents?

Case 1. Girl, 7 years old, grade III, I. Q. 88.

P. Twenty naughts are zero, so twenty threes are 60 (a long pause).
O. What are you studying about?
P. 20×30. (While she was writing her 30's, she said: "I will just have to add ten 30's) I'll know 2 times that will be it." She added ten thirties and multiplied the sum by two and said, "Cost $6.00." She did not know how to multiply when both numbers ended in zero.

B. Joseph rode on the merry-go-round 12 times. Each time it cost him 3 cents. How much did he pay for all the rides?

Case 1. Boy 7 years old, II-A.

P. 36 (after considerable thinking).
O. What?
P. Three 2's are 6 and 3 are 9 and 3 are 12 and 3 are 15, etc., until 33 and 3 are 36.

Other children added twelve three times instead of three twelve times.

C. I had 24 apples and divided them equally among 4 boys. How many apples did each boy receive?

Case 1. Girl, 8 years old, IV-B, I. Q. 74.

P. 0000 0000 0000 0000 0000 0000.

IV. Dull children try any process until teacher's approval is met.

A. When John was 13 years old he received $60.00 each year in monthly payments for his spending money. How much did he receive each month?

Case 1. Boy, 13 years old, grade V-B.

P. (Reads problem).
O. What are you going to do there?
P. Subtract.
O. Go ahead.
P. $60.00 — 13 = 40.97.
O. Why did you subtract?
P. It is the only way I can see how that should be done. It would sound much better to subtract.
O. What did you subtract?
P. 13 from 8.
O. Think that is the way you ought to do it? $40.97 a month looks like too much for $60.00 a year.
P. $60.00 \times 13 = $180.00. That makes all the more. There is just one more way left—add. I can't see where you add this very easy.
O. Think that is wrong?
P. $60.00 + 13 = $60.13.
O. What are you trying to find out?
P. What he received each month. This is $60.13
O. That is more than he received the whole year, isn't it?
P. You can divide by 13. 13)$60.00.
O. Yes.
P. Maybe it would come out better—with a smaller amount. Now, before you divide by 13, read the problem out loud. (Reads the problem).
O. What do you want to find out?
P. Find out how much he received each month.
O. All right. How do you think you would do it? How many months in a year?
P. 12.
(Pupil did not attempt to use the 12 in solving problem).

B. What will 20 marbles cost if you can buy 2 for 1 cent?

Case 1. Boy, 12 years old, V-B.

P. Reads aloud then begins to solve it thus) 20 — 21 =
O. Can you do that?
P. (tried division). 20)21
O. Read the problem again. What does it tell you?
P. Two for one cent.
O. What does it ask you?
P. What will 20 marbles cost?
(Pupil then tried addition and multiplication.)

V. Bright children solve part or all, mentally.

A. A boy had 210 marbles. He lost one-third of them. How many were left?

Case 1. Boy, 11 years old, grade VI-A, I. Q. 117.

P. (Solves thus) 210
— 70
140

(Pupil found one-third mentally and subtracted.)

B. George walks at the rate of 2½ miles per hour. Henry starts with him and walks in the same direction at the rate of 3 miles an hour. How many miles apart will they be in 3 hours?

P. (Thinking but making no marks) 2 miles.

SUMMARY

A study of the methods used by pupils in solving problems shows at least five methods of attack:

1. The size of the numbers and the wording of the problem help them to decide what process to use.
2. Many pupils try to do something with all the numbers.
3. Pupils not knowing the fundamentals attack the problems in peculiar ways.
4. Dull children try the different processes until the teacher's approval is obtained.

Problems Without Numbers

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AND
VANCE M. SMITH

A problem is a situation coming naturally into the life or experience of an individual and capable of arousing his interest in its solution. Practical life abounds with problems of every kind and yet as a rule the child thinks of a problem as something in an arithmetic book. He must be taught to see that many problems are without numbers.

Problems without numbers break up the habit of starting with the given figures in the problem and not thinking clearly through it step by step. These problems, therefore, lead to better thinking and a more orderly way of handling the material given. Since there are no numbers to distract attention, it is easier for the children to attend to the following logical steps: (1) What is the problem about? (2) What is asked? (3) What facts are given? and (4) What process or processes are used?

Problems without numbers should be used first in the third grade and then through the succeeding grades until the pupils have formed correct habits of problem solving.

In such work, emphasis should be laid on getting the meaning of the problem rather than on the manipulation of figures. From third-grade children no detailed explanation should be expected. Simply require the pupil to state which of the four fundamental operations he used in the solution of the problem. He might say, "I added the three things that were given," or "I multiplied them," etc. The problems should always be within the realm of the child's experience if he is to get the thought and express himself in arithmetical terms. At the beginning of this work, it is a good plan to choose pupils who express themselves well, thus furnishing good models for those who need help. Dramatization may be effectively used in developing these problems, especially in the third grade and the first half of the fourth.

By the time the pupil reaches the upper grades, he should be able to find problems in his everyday expe-

rience, whether at home, on the way to school, doing errands, or in more formal arithmetic work. In the upper grades the method of teaching problems without numbers should be varied. Either read the problems to the pupils or write them on the board and have them analyzed according to the steps already given. It is wise to change the order of the second and third steps so that the child will not be lost if he meets the situation reversed.

It is hardly necessary to state that from the fourth grade on the problems should contain both one and two steps involving each of the different processes. As in all other teaching, great care must be exercised in this work. It will be better to use only a few at a time on frequent occasions, rather than to give long lists of exercises.

We have attempted to show the results of using problems without numbers by setting forth their value in destroying incorrect habits of problem solving; in teaching children to handle material in a more orderly manner; and in teaching the use of steps in problem solving. These problems should be introduced in the third grade and their use continued until the pupil has formed correct habits of solving them.

The use of this sort of problems stresses the interpretation of the situation rather than the manipulation of figures. This work shows that problems should be varied according to the child's experience and the number of steps in the problem, and that the manner of presentation should also be frequently changed. This kind of work will, I am sure, prove a valuable preliminary to solving problems with numbers.

In order to help those who may be interested in this method we are giving a graded list of problems without numbers, from which others may be derived.

THIRD GRADE

1. If you knew how much John put in the bank each week in February, how could you find the amount deposited in the month?

2. John drew out some of his money in May. How can you tell how much he still has in the bank?

3. If you know the cost of one pound of sugar, how could you find the cost of a number of pounds of sugar?

4. If you knew the amount paid for a ton of coal, how could you find the cost of one-half ton?

5. In this room there are so many seats in a row. How could you find how many seats there are in several rows?

6. If you knew Alice brought so many pennies for milk and lost so many, how could you find the number of pennies she had left?

7. If you know the cost of three different things, tell how to find the cost of all of them.

8. James divided his marbles equally among his brothers. Can you tell how to find the number of marbles each would receive?

9. If you know your father's age, how would you tell how much younger you are than he?

10. If we know the weight of each child in our room, how can you find the weight of all the children?

FOURTH GRADE

1. The fourth-grade class went on a picnic. The lunch cost a certain number of dollars. Tell us how to find what it will cost each pupil.

2. If you knew the number of people living in Columbus and in Cleveland, how would you find which is the larger city?

3. If we know the cost of one history book, how will we find what the books for the class cost?

4. If you knew the length and width of the schoolroom, how would you find the perimeter? Area?

5. Jane pays so much for a reader, an arithmetic, and a geography. She gives the clerk more than they cost. How can you tell what change she would receive?

6. Robert paid the car fare for his two sisters, his brother, and himself. How can you find how much money he spent?

7. If newspapers cost a certain amount of money, and are sold at a profit, how will you find how much money is made on the papers bought?

8. If you knew how many yards of gingham were bought for a certain number of dresses, how could you find the number of yards used for one dress?

9. Mr. Brown pays a certain sum for rent each year. How much is the rent for a month? For a number of months?

FIFTH GRADE

1. If you knew the selling price of one-half peck of potatoes, how would you find the price of a bushel of potatoes?

2. If you had all your test grades for a certain length of time, how could you find your average grade?

3. How do you find the balance on hand, if you have the amount received and the amount paid out during the year?

4. James had some rabbits and sold part of them. How can you find how many he had left?

5. If you knew how much a man's income was for a month and how much he spent during the year, how would you find out how much he saved in a year?

6. If you know how many feet long the schoolroom is, how would you find how many yards long it is?

7. The farmer has a large pail of milk. He knows how many quarts it holds. How can he find how many pint bottles he can fill from this pail?

8. How can you find the cost of a number of quarts of milk when you know the price of one quart?

9. John sold some of his rabbits. Then he bought some more. How can you find how many he then had?

SIXTH GRADE

1. If you know the number of decimal places in each of two numbers, how do you find the number of decimal places in their product?

2. If a profit is represented in money and also in percent, how may the cost be found?

3. If you knew the marked price on a certain piece of furniture and the percent of discount given for cash, how would you find the amount saved by paying cash for the furniture?

4. Given the area and the length of a rectangular lot, how could you find the width?

5. If you knew the principal of a note, the rate percent and the length of time it ran, how would you find the interest?

6. How would you find the area of a field in acres, if the dimensions were given in rods?

7. If the dividend of a problem has four decimal places and the divisor has one decimal place, how do you know how many places to point off in the quotient?

8. If you knew the cost of a certain number of apples, how would you find the cost of a smaller number of apples? How would you find the cost of a larger number of apples?

9. If you know what a given fraction of a number is, how do you find the number?

10. If you know the selling price of a certain article and the amount of loss, how may you find the percent of loss?

Vocabulary Difficulties

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AND
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This report proposes to discover what are the so-called "vocabulary difficulties" in solving arithmetic problems, and whether they are real or imaginary.

Investigation has proved that such difficulties are real and are always present, they hinder the pupils in their understanding of the full import of the problems to be solved. The words and phrases used are often strictly mathematical and bear a sense peculiar to the subject. When pupils fail to interpret them correctly, they become a source of difficulty.

The purpose of arithmetic is to enable one to solve the problems of practical life. Consequently the language of the problems should be clear, simple, and attractive, devoid of terms and phrases that are too difficult and technical for the pupils who are expected to solve them. If unfamiliar and technical terms must be used,

however, they should be thoroughly explained to the pupils before the solution of the problems is attempted. Further, not only words and phrases must be explained, but the various signs, symbols, and abbreviations which are also part of the language of arithmetic problems should be given their due share in the explanation.

Investigations were made to acquaint ourselves at first hand with the more common words which give trouble. Thirty-two teachers of several buildings including elementary and junior high-school grades were each asked to bring in about fifty words which in their opinion gave trouble to pupils. Twenty-five teachers sent in a total of 1,250 words. These words were tabulated. Three hundred ninety different words, having a range of frequency of 1 to 21, were listed. The following table gives words that occurred 10 or more times:

dividend	21
area, average, minuend	20
denominator, integer	19
product, quotient, subtrahend, numerator	17
notation, profit, multiplicand	15
multiplier	14
subtrahend	13
divisor, estimate, ratio, compute, rate, fraction	12
amount	11
aliquot, reduction, value, discount, difference, increase, interest	10

Some of these words were then given to more than 800 children, they were asked to give the meaning of these words when used in arithmetic problems. The results of the answers from the several pupils proved conclusively that a large percent of the pupils did not understand many of the terms used in the problems.

Some strange answers were given:

Quotient—

The quotient is a mark, $\begin{array}{r} 3. \\ \hline .41)609 \end{array}$

Is the divisor (2) $\begin{array}{r} 3 \\ \hline)6 \end{array}$

Means the sum.
Answer in subtraction.
You get a quotient when you add two numbers.
The number from which you subtract.
The number left.

Ratio—

The answer to subtraction problem, 124—24=100 ratio.
Ratio is distance between, the sun is 100,000 miles to earth.
The distance around a thing.
Interest is called ratio.
The rate by which money is loaned.

Product—

Answer to subtraction, 121—21=100 product.
The proof of subtraction.

$$\begin{array}{r} 21 \\ 10 \\ \hline 11 \\ \hline 21 \end{array}$$

Is what you gain.
The answer in subtraction.
When 2 or more numbers are added.
How much you get when you add.
Is when you subtract.
The number left over when you divide.
Answer in addition.
Is the remainder.
What you raise in a country.

Total—

The answer in multiplication.
The amount paid.
How much you have left.
When you divide two numbers you get the total.
Is what is left over.
Is the area.
Is the cost.
Means a lot of money.
The amount left in dollars and cents, $2.50—1.00=$8.50 total.

As another means of studying vocabulary difficulties, a list of fifty words commonly used in arithmetic problems was prepared and checked with the *Teacher's Word Book*[1] which contains a list of ten thousand words which are used most frequently in common forms of literature. These words are arranged in groups of one thousand in

(1) The *Teacher's Word Book* is an alphabetical list of 10,000 words which were found to occur most widely in a count of 625,000 words from literature for children; about 3,000,000 words from the Bible and English classics; about 50,000 words from books about cooking, sewing, farming, the trades, and the like; about 300,000 words from elementary-school textbooks; about 90,000 words from the daily newspapers; and about 500,000 words from correspondence. Forty-one different sources were used.

numerical succession according to the frequency of use, that is the words in the "First One Thousand" occurred most frequently in the material upon which the list was based, while the second, third, fourth, and fifth occurred in correspondingly decreasing frequency. Upon checking these fifty arithmetic terms it was found that the majority fell within the first five "One Thousand" group, while eleven words were so unusual as not to be included within the *Word Book*. These fifty words were grouped as follows:

First Thousand: account, amount, bill, check, cost, distance, figure, gain, half, interest, loss, measure, note, part, pay, price, pound, price, value.
Second Thousand: acre, dozen, earn, group, product, profit, rate, surface, term.
Third Thousand: area, bank, cash, commission, peck, quart, receipt, salary.
Fourth Thousand: pint
Fifth Thousand: fraction, gallon.
Not listed at all: average, change, cubic, degree, denominator, deposited, dimension, discount, expenses, oblong, unit.

As a means of corrective and remedial measures, pupils were asked to read several problems and to make a note of the words which they did not understand; these words were explained by some other pupils or by the teacher. Pupils were then asked to re-state the problems using words which gave them a freer and easier understanding of the problem. This exercise revealed many words and phrases which were a constant source of difficulty to the pupils.

The following problems were read by the teacher to the pupils of a V-A class:

What is the increase in a boy scout's walking record, if on a tramp he extends his record 8½ miles to one of 12 miles?

Because of the blank expressions on the faces of the majority of the pupils, the teacher asked whether there were any words in the problem that they did not understand. Some did not understand "increase" so the teacher called upon a pupil to explain, and he said "how much more." "Tramp" was next called for; one pupil said that it meant the walk the boy scouts took, another replied that when boy scouts walk they call it a "hike." One child was puzzled by the phrase "to one of 12." With these explanations the pupils set to work and 60 percent of them solved it correctly.

A lake is 2 ¾ miles across. How far from the farther shore is a canoe which has been paddled 1¾ miles across the lake?

After the term "farther shore" was explained the results were quite satisfactory.

How much greater is the distance covered by a boy who swims ½ mile at a stretch than that covered by another boy who swims 220 yards (⅛ of a mile)?

In this case "stretch" was the obstacle; after this was explained, all went well.

A laundress worked at 20 cents per hour, from 8 o'clock in the morning until 5 o'clock in the afternoon; how much was due her if no reduction was made for noon hour?

After the pupils got the meaning of "laundress" and "reduction" few had any difficulty.

The findings of our investigation have fully convinced us that technical words and phrases are sources of great difficulty and hindrance to pupils in the solution of arithmetic problems, hence to improve the standard of the work of the class in the solution of problems, the teacher must pay attention to this phase of the work. She must give the proper explanation when and where needed. This in our opinion will reduce the mistakes to a marked degree, and raise to a higher level the pupils' ability in solving problems. Not only will this be a great satisfaction to the pupils themselves, but will also serve as an ample reward to the teacher.

Variety in Problems

ETHEL S. COLE
AND
EMMA J. COOK

Problems for children should be varied and should resemble actual conditions. Heretofore most problems have been unreal and have failed to stimulate any thought. A large and varied number of problems should be given to pupils so that they may be able to think out real problems for themselves and apply arithmetic to all kinds of situations. Problems should also be practical not only in nature but in figures; if butter is given at thirty-five cents a pound or shoes at two dollars a pair when butter is sixty cents and shoes are six dollars, the teacher and pupil should change the figures.

Arithmetic makes a very strong appeal to children because of their interest in mental activity and achievement. They are interested when the problems reflect their experiences in school, at home, at play, and in other activities with which they are familiar. All children prefer definite tasks, for then they know what they have to do and when it is done; they enjoy the sense of action, achievement, and mastery. In arithmetic problems, therefore, language must be considered; it must be free from useless difficulty. The newer methods insist that the textbook and teacher should preserve the pupil's healthful interest in arithmetic by eliminating useless difficulties of vocabulary or construction.

Problems should be stated in different ways so that the pupil will get no clue as to the process from the wording. Pupils frequently jump at the process that seems to be indicated by the wording, as in the example when the word "times" was used and the pupil multiplied on that account[1].

(1) See p. 275 of this issue.

The teacher should, first, try to provide real situations, and second, encourage the pupil to identify himself with the person who is represented as acting or planning. In problems which do not lend themselves to such treatment the teacher should endeavor by explanation to rid the problem of any difficulties of vocabulary or structure and to explain unfamiliar circumstances.

In order to base problems on the actual interests of the pupils the author spent considerable time and thought in making out a list of pupils' activities and interests. These included a camping trip, a school picnic, a birthday party, purchasing an automobile, Christmas shopping, a visit to a furniture factory, savings, selling papers, planning trips, a school field meet, and going to the circus.

Since the present arithmetic texts contain a limited number of interesting problems a teacher will find it worth while to keep, from year to year, a number of problems applied to different topics. A loose-leaf notebook or filing cards can be used for this purpose. The pupils can help the teacher in devising problems such as the following, which were worked out by the pupils and teachers in a fourth grade.

THE CIRCUS

1. John earns twenty-five cents a day. How long will it take him to earn enough to buy a seventy-five cent ticket to the circus?
2. Sarah wishes to buy a seventy-five cent ticket to the circus. She already has forty cents. How much more does she need?
3. James spent seventy-five cents for a circus ticket, ten cents for pea-

nuts, and five cents for candy. How much did he spend in all?

4. If the circus performance commences at 2 o'clock and ends at 4:30, how long will it last?

5. If lemonade is five cents a glass, what will be the cost of four glasses?

6. The weight of the bear is 960 pounds and of the giraffe 1,472 pounds. How much more does the giraffe weigh than the bear?

7. The amount received at the circus on Monday was $672.98, on Wednesday, $812.10, and on Saturday $1,265.47. How much was received for the week?

8. One cage has 16 monkeys in it. How many monkeys are there in three such cages?

9. A boy earned $1.50 a day, carrying water to the animals. How much does he earn in 6 days?

10. The distance around the circus rings was 450 yards, 680 yards, and 720 yards. How far has a horse traveled to go around all three rings?

11. There were 1,246 tickets sold the first day. If one ticket cost seventy-five cents, what did they all cost?

12. The amount received by a circus at Dayton was $798 and at Columbus $926. How much more was received at Columbus than at Dayton?

13. There were ten lions, fourteen elephants, eight camels, twelve bears, twenty seals, and seventy-seven horses. How many animals were there in the circus?

14. If the clown gets $150 a week and the trapeze performer $285 a week, what do they both receive?

15. The expenses of a circus were one-third the amount received. If the receipts were $1,800, what were the expenses?

Pupils are able to bring in actual problems from home of which the following are illustrations:

1. My uncle bought a car for $1,100. He paid $500 down and expects to pay $30 per month on it. How many months will it take him to pay for the car?

2. My mother owns twelve garages, for each of which she gets $5.00 per month, and a double house which brings in $110 per month. How much is my mother's income each year from these properties?

Methods of Attacking Problems

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AND
FANNY M. KIGHT

When left to their own devises pupils tend to start doing something before they really know what the problem is or have any definite plan for its solution. The following scheme has been suggested and has proved very effective. Have the children answer these four questions before actually starting to work the problem: (1) What question is asked in the problem? (2) What facts are given in the problem? (3) What process or processes should be used in solving the problem? and (4) What is a reasonable answer? In looking over the four points just given it is quite plain that the first question, What is asked in my problem? is of no little importance, since

neither the third nor the fourth question can be answered until it is fully determined.

During the past winter the Stevenson Problem Analysis Test was given to a group of seventh- and eighth-grade pupils. For observation let us choose three of these papers with scores ranging from 8 to 23 points out of a possible 24. One problem read:

How much would 10 automobiles cost a dealer at $780.00 apiece?

The pupils were asked which of the following four facts were to be found:

1. The gain on one automobile.
2. The kind of automobiles the dealer bought.
3. The cost of all the automobiles.
4. The amount the dealer received for each automobile.

One pupil said number four, the amount the dealer received for each automobile; others said number three, the cost of all the automobiles. After checking and discussing the papers the first pupil was asked if the amount the dealer received for each automobile was given. She replied that the $780 was the amount the dealer received for each automobile. She was then asked whether the automobiles were bought or sold. Immediately she saw her mistake. It is evident, then, that the child does not always understand at once what is asked for in the problem.

In still another problem, 720 fancy apples are put up in sample boxes of five apples each. How many fancy boxes will it take to hold the apples? Again the children were asked which of the following facts were asked:

1. The number of bushels of apples put up in the boxes.
2. The number of boxes it will take to hold all the apples.
3. The number of apples put in each box.
4. The amount of money received from the sale of the apples.

A marked No. 3 as the fact required. B said No. 1 and C, No. 2. Again after a little discussion both A and B saw their mistakes. The children were drilled on this type of problem, and several weeks later when a second problem analysis test was given, A raised her score from 8 to 17 points, B from 11 to 15, and C had a perfect paper.

A small amount of drill is very effective in teaching pupils to comprehend what is wanted in problems. Have the pupils read the question *as given in the book* and demand that *not* the *entire* problem, but only the *question*, be read. The question should also be stated in as many different ways as possible.

The second step in solving a problem is to find out the facts which must be used in working the problem. The problems should involve as far as possible actual situations in the child's life. Life problems are most often questions about situations or facts actually existing within the child's experience, less frequently questions which the child puts to himself in relation to his everyday life, and least often questions put to him in words by another.

The analysis should be an orderly, concise statement of thought steps. Some of the questions the child should ask himself are: "What facts are given in the problem?" "How shall I use these facts?" and "What shall I do with the numbers and which numbers shall I use?" By means of these questions the child will select the numbers which he must use, instead of writing the first number in the problem and then wondering which number to write next.

This problem was given: "A girl had 240 chickens and ¼ of them died; how many did she have left?" One girl in working the problem said, "The girl had 240 chickens and ¼ of them died. There were sixty chickens that died, and so I must subtract that from 240 to find how many chickens were left."

In another instance, where there was a number not to be used, a boy started out in a different way. The problem was: "Six boys went on a camping trip. They stayed two months, and the cost of the trip was $68.40. How much did each boy have to pay?" He read the problem through and said, "I don't need the two months. If there were six boys on this camping trip and it cost all of them $68.40, then 6 and $68.40 are the numbers to use. Divide $68.40 by 6 and the answer will be the cost for each boy." Pupils should be given problems containing facts not needed in the solution because it will then be necessary for them to select the essential facts. Such selection is necessary in many problems in real life.

If a child has answered successfully the question of what is given and the problem is based on his experience, he should have little difficulty in choosing the process. It helps a pupil to decide on the process, if he himself has been accustomed to formulating problems.

Pupils have been helped by the following method: Select a list of one-step problems in which the numbers are simple. The pupils should plan the solutions without working out the problems. It is obvious that the habit of putting down figures before thinking out the process may thus be broken up. By a reasonable amount of such exercises the ability to select the process may be increased. The difficulty of the problems may gradually be increased from one-step to two-step problems. They should be made very simple so as not to discourage the pupils.

The last step, and not the least important in solving problems, is estimating answers. Teaching children to estimate answers is of great value in many ways. Perhaps a teacher appreciates most the fact that children do not give such wild answers. They very quickly see how ridiculous some of their answers are. In a VI-A class this step was emphasized to such a degree that in percentage problems the children could at once tell whether the answer should be larger or smaller than the given number. For example, if 10% or 12½% of a number were 15, 100%—or the number in question—would be a great deal larger than 15; and if 75% or 87½% or 90% of a number were 63, then 100%, or the number itself, would not be much larger than 63; and if 125% or 150% were 30, then 100% would be smaller than 30.

Estimating answers helps overcome the tendency to work mechanically. In many businesses one is called upon to give a quick estimate as to the cost or the amount of certain things, and if one has been properly drilled in giving estimates this can become a habit. Children *can* be taught to estimate answers, but it must be done gradually, since it takes time to teach them to think in round numbers when addition, subtraction, multiplication, and division have been taught so definitely. Start with simple one-step problems such as: "One chair cost $2.75; find the cost of 5 chairs at the same price." Some pupils will readily see that the answer will be about $15 and some will tell you that your answer will be between $10 and $15. Those who do not see how these answers are obtained must be shown by being questioned and led on, one step at a time. After some work along this line children seem to like it.

In a VI-A class the following example was given: "Find the cost of 5 chairs if 2 cost $4.75." One of the boys estimated his answer this way: "2 cost $4.75; 4 cost around $10; 6 cost near $15; so my answer will be somewhere between $10 and $15."

In a VI-B class that had had very little drill in estimating answers, this problem was given: "The distance from Chicago to Cairo is 363.57 miles; from Cairo to Memphis is 170.4 miles; and from Memphis to New Orleans is 395.38 miles. What is the entire dis-

tance from Chicago to New Orleans?" When asked to estimate the answer, most of the pupils wanted merely to drop the decimals, which would not help much. They were then told to add the hundreds—300, 100, and 400. Next the teacher asked how many thought that the second should be taken as 200 instead of 100, and why they thought so. Some quickly saw that 170 was nearer 200 than 100 and that it evened up the other two. Finally they decided that their answer would be between 900 and 1,000 miles. Then when they worked the problem all but four of them had it correct. These four added 300 and 200 and 400 instead of the right addends. This is the one difficulty encountered at first; some pupils wanted to give the estimated answer for the correct one.

Estimating answers in decimals is a great help, for children never seem to realize that a decimal fraction is a part of a unit just as a common fraction is. In problems using U. S. money, some children are never certain where to put the decimal point. For example, this problem was given in a VI-A class: "I put 30% of all my money in the savings bank. If I put $60 in the bank, how much money did I have?" One girl worked it as follows:

30% or 3/10 is $60.00; 1/10 is (1/3 of $60.00) $20.00; 100% or 10/10 or all my money is $2.00. All the teacher had to say was, "How could I put $60.00 in the bank if I only had $2.00?" The girl saw immediately that she did not need the decimal point.

A little drill each day or several times a week in these four steps of problem solving are very helpful in teaching children to help themselves in this work.

Dramatization in Arithmetic

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Since activities mean much more to the child than abstract ideas, dramatization helps children in solving arithmetic problems. Through his own activities interest is aroused. He learns more, if interested, because he enters wholeheartedly into the activity. If he is especially interested in his work, he will make an effort to obtain good results.

Dramatization has socializing value for it gives training in cooperative work. In a fourth grade the children were encouraged to find current prices of various articles used. With this knowledge the class made out charts for use in a play store. The storekeeper made out on regular bill forms, accounts for all goods sold. The purchasers were taught to calculate the amount of their bills so as to detect any errors on the part of the storekeeper. Since the store handled articles sold by dry measure, that part of the work was dramatized. Beans, corn, etc., were used for this purpose. The children were thus made familiar with the actual use of the quart, peck, and bushel measures.

Dramatization of arithmetic may be easily correlated with such other subjects as, hygiene and health, geography, and English. In the correlation with hygiene and health, the class

which receives milk daily may figure how many pints, quarts, and even gallons their room uses.

Dramatization makes the child see purpose or value of arithmetic work, since it provides for the application of knowledge learned. In teaching area, dramatization was used very effectively. The teacher cut out a number of cardboard squares, each edge of which was one foot. These were passed round to different children who measured them. The class thus gained a clear idea of a square foot. One child was sent to measure the length and height of a certain blackboard. These measurements were marked down, then different children were asked to place their square boards side by side along the base of the blackboard. The children compared the number of square feet in one row with the length of the board. The teacher asked, "What surface is covered by one of the squares? by all the squares in this row?" A child measured the height of one row. Then other children were called to place their squares in a second row, directly above those of the first row; a third and fourth row of squares were set up. The board was now covered. The children compared the numbers with the height of the board. The children noted that there were the same number of square feet in each row; and they saw how many square feet were required to cover the board. In similar manner the areas of various rectangles drawn on the floor were measured.

In the sixth grade where ratio and percentage are the important topics, these phases were developed through business dramatization. Children were asked to bring in newspaper advertisements which would furnish materials for this work. A great amount was gathered in this way for use in ratio. Signs such as "Apples, two pounds for fifteen cents," "Three oranges for ten cents," were made and used in the store. If a child wanted eight pounds of apples at two pounds for fifteen cents the storekeeper and purchaser figured the price.

Through the furniture business, discount was made more real. The boys went to a neighboring store to get facts concerning articles, and discounts allowed for cash payments. A bank and a building and loan company were also established. The children are impressed with the importance of saving at least a portion of their earnings. The children have practice in making deposit slips, in writing and indorsing checks.

Real-estate business has also been dramatized in this grade. One boy having a relative who is a real-estate dealer secured all the necessary information. The children were told to clip from the newspaper any real-estate advertisements which appealed to them. Those bringing in such advertisements played they owned the property and listed it with one of the class agents, who advertised it. When the property was sold the owner paid the agent the commission, by means of a check. The purchaser made his cash payment and went to the building and loan for the balance. According to the actual methods used by the building and loan companies, the children figured the monthly payments to be made on a particular piece of property, the interest on the principal for six months (that being the time decided upon), and the amount of money left from the six monthly payments after the interest was deducted was applied to the principal.

Grade VI-A takes complete charge of the school banking for the entire building. A teller and assistant were appointed for each class. The bank is in operation once a week from 7:50 to 8:30 A. M. The depositors make their deposits with the tellers chosen for their respective rooms. The teller

makes out the deposit slips, and also marks in a book the amount of each deposit. At the close of the banking time, the assistant balances up the book, while the teller adds up the amounts on the deposit slips, and counts the money. All sums must tally. Then the teller makes an itemized statement of the money taken in. Finally the members of the class figure the percent of the pupils in each room who deposit.

Assignment Lesson on Arithmetic Problems

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For many decades, many teachers of mathematics have used the entire class-time in having pupils copy on the board problems which they have "worked" at home the night before and in having the solutions read from the board, as a so-called explanation. After the closing bell, the teacher hurriedly assigned new home work by saying, "For tomorrow take the first fifteen problems on page 167."

This plan of conducting the recitation is, no doubt, responsible for much of the ill repute of mathematics and for the mistaken notions concerning it. The teacher did no teaching, that was left to fathers, mothers, aunts, older brothers and sisters, or even to kind neighbors. The teacher became the official tester only. Progress was slow, for solutions brought from home and copies on the board did not represent the pupil's work. He was, therefore, unprepared for subsequent topics, which became unnecessarily hard.

In more recent years, since psychologists have studied and directed our attention to the learning and teaching processes, a complete reversal of the plan has come about. The teacher, specifically trained for teaching, has learned that all new work must be presented by him in class; that home work should be practice work or testing.

With this change in procedure, the teacher has come to realize the great importance of the assignment and to give it its proper place in the recitation. It is no longer left to the last moment and limited to "Take the next ten problems." It may be given first, in the middle of the period, or towards the close of the arithmetic period, according to the type of lesson, but it is always carefully thought out, clearly and definitely presented, and never hurried, for it is usually the most important part of the recitation.

Among the functions of the assignment is that of making the pupil feel a real need for learning the process. It should create such an atmosphere that he becomes eager to do the task or learn the lesson. If there are any particular difficulties to be encountered, he should be given a chance to start the work in class until he comes to a difficulty, when he may be aided by a question or a suggestion.

It is much better to consider the classroom as a laboratory or a workshop, where pupils do their learning of new material. The home work should be material previously learned and thoroughly understood, it should include processes on which practice is needed for the development of necessary skill and feeling of mastery.

Many a pupil has been heard to say, "I can do the computation all right, but I can't do those worded problems." I wonder why. In arithmetic there are a very few fundamental processes which must be learned for both integers and fractions. Surely these are not beyond the capacity of any normal boy or girl, if the numbers used are limited to those of reasonable size or difficulty. No others should be taught, for they are not needed.

The solution of a problem, however, is something more than computation. Its difficulty is its logic, not its mathematics. In computation the pupil is taught a definite plan of procedure. He is taught how to attack or how to "set down" an exercise in division, but in a worded problem he does not know how to start. An assignment of problems is incomplete that has not given the pupil some method of attack.

One suggested plan of procedure is as follows:

1. Have the pupils read the entire problem carefully. In some cases this should be repeated, for often mere deficiency in reading ability is the real cause for failure in mathematics.
2. Have the pupil re-read the problem in order to select the given data and to state what he is to find.
3. The third step is to consider what process or processes to use. Perhaps, a wrong suggestion is made, but the following out of a wrong suggestion to a recognized illogical conclusion is more educative than drawing a correct conclusion unwittingly from a correct suggestion given by another. If a pupil develops the habit of making estimates and the ability to judge them as reasonable or unreasonable, he has a valuable aid in making the correct choice of process and a check on a wrong choice.
4. He should then apply the selected process and get an accurate result, which should be compared with his estimate.
5. By applying more accurate checks, he determines for himself whether the solution of the problem is correct or not.

To develop the ability to select the processes, much practice should be given in problems without numbers. The mere getting of an answer is an unimportant part of this thinking. Most pupils overestimate its value.

Practice in selecting data may be given by having pupils read a list of problems and state exactly what is given and what is to be found. Then in a supervised-study period have each one decide for himself what processes should be used and make his estimates. If time is lacking, the actual computation may be assigned as home work.

It is difficult to make an interesting assignment with uninteresting types of problems. The success of an assignment, therefore, depends largely on the selection of material. Practical problems should be used as far as the maturity and previous experience of pupils permit. The problems should be "real" to the pupils. The fact that an adult in some walks of life may actually have to solve a certain problem does not guarantee that it will be real to the child. Too long have our texts been filled with various types of artificial problems, as the clock problem, the cistern problems, and the A-B-and-C work problems.

Other problems which are not interesting to pupils are those which have absurd numbers, as the following taken from the text of a prominent university professor:

> If a motor car is travelling at the rate of 90 miles per hour, how many feet is this per second? How does this speed compare with a bullet from a rifle which goes 2,400 feet per second?

This illustration shows also that more attention should be given to the reasonableness of the content. Many pupils are very keen in detecting which problems have unreasonable or incorrect data.

Greater interest is taken by pupils in a lesson assignment which has continuity. Instead of assigning a miscellaneous set of disconnected problems, it is better to give a list of related

ones as illustrated by the following problem:

1. A man wished to buy a lot for $3,000. He borrowed $500. He paid 6% interest on it for 120 days. What was the real cost of the lot?
2. In one year he sold the lot for $4,000. How much did he gain if taxes for the year were $45?
3. If, instead of buying the lot, he had lent the $3,000 at 6% for the year, how much interest would he have received?
4. How much more did he earn by buying and selling the lot?
5. Suppose he had not been able to sell the lot for five years, and then received $4,000 for it. Would he then have made more money by buying the lot or by lending the $3,000 for 5 years at 6%?

A good practice is to assign to pupils the task of making up problems. More real ability is developed in the formulation of twenty problems than in the mere solution of twice as many. The problems submitted by pupils are usually practical and real to them, for they come from their own experience. The campaign of teachers of arithmetic against the artificial and absurd types of problems will be successful, when authors eliminate them from their textbooks.

As an illustration of a lesson in percentage, the following types of problems are offered, which are simple, practical, and real to boys and girls. The aim is to teach the process of finding what percent one number is of another. The abstract terms, "base," "rate," and "percentage" are not used, because mathematicians in general have discarded them, and rightly so, nor are they now used in business practice. A merchant may speak of the cost, his rate of interest, or his gain but he never uses the abstract terms.

The use of the process may be shown by such a question as:

Is it more expensive for a shoe-dealer to put an 8 cent lace in a $12 pair of shoes or to put a 5 cent lace in a $6 pair?

This question also gives the child some idea of how the successful business man must always think in terms of percent.

The personal feeling of need for learning the process may be created by such questions as:

If an arithmetic lesson has 10 problems of which you solved 7 correctly, what part of the lesson was correct? What percent should it be marked?
How many pupils are in your class?
How many of them are boys?
How many are girls?
What part of your class is boys?
What percent of your class is boys?
How many basketball games has your team played this year?
How many have they won?
What percent of the games have they won?
What is the estimated population of your city at present?
What was its population at the last census?
What is the increase since the last census?
What is the percent of increase over the population at the last census?

Such simple and ordinary questions should be continued till the topic is developed and the pupil feels that he thoroughly understands the process.

Applications to give further practice may resemble the following:

If a school boy is allowed $1.50 a week, and he plans that he must spend 75 cents for lunches, 15 cents for pencils, pens, and paper, 25 cents for car fare, and finds he can use 20 cents for recreation and save 15 cents, he is making a personal weekly budget.

Why is it wise for every one to make a budget?

What percent of his allowance does he spend for lunches? for pencils, pens, and paper? for car fare? for recreation? What percent does he save?

or

Mr. White has an income of $900 a year and spends $360 on food. What percent does he spend on food?

Mr. Black has an income of $1,800 a year and spends $630 on food. What percent does he spend on food?

Mr. Brown has an income of $2,400 a year and spends $600 on food. Compare the percent Mr. Brown spends on food with that of Mr. White and Mr. Black.

How would you find out what percent your mother spends on food?

or

Mrs. Green has an income of $40 a week. After all her expenses are paid she is able to save $7. What percent does she save?

or

A boy buys newspapers at 1¼ cents and sells them at 2 cents each. What percent of the cost does he gain? What percent of the selling price does he receive?

or

During a special sale a store offered $2 silk material for $1.59 a yard. What percent was saved?

or

After Easter a boy's suit was reduced from $25 to $19.75. What percent was saved by waiting?

or

A girl's hat was reduced from $10 to $6. What percent did she save?

By this time the class should be ready for such an assignment as:

1. During the war prices of food, clothing, and other commodities were greatly increased. Find the amount and percent that each of the articles listed below were increased.

2. Even in peace-times prices change greatly from year to year. Compare the present prices with those of 1919, and find the percent of increase or decrease of each.

	Article	Cost in 1913	Cost in 1919	Amount of Increase	Percent of Increase
1	Bread	.10 a lb. loaf	.15 a lb. loaf		
2	Butter	.35 a lb.	.75 a lb.		
3	Eggs	.36 a doz.	.78 a doz.		
4	Ham	.30 a lb.	.60 a lb.		
5	Lard	.12 a lb.	.36 a lb.		
6	Potatoes	1.00 a bu.	2.40 a bu.		
7	Sugar	.06¼ a lb.	.11 a lb.		
8	Pork	.18 a lb.	.40 a lb.		
9	Flour	1.00 a bbl. sack	1.80 a bbl. sack		
10	Milk	.08 a qt.	.15 a qt.		
11	Coal	3.25 a ton	6.50 a ton		
12	Shoes	8.00 a pair	12.00 a pair		
13	Jersey cloth	4.00 a yard	6.50 a yard		
14	Silk	1.50 a yard	2.50 a yard		
15	Serge	1.25 a yard	2.25 a yard		
16	Silk gloves	1.00 a pair	1.75 a pair		

In this presentation of the worded problem, first the use is shown; then a feeling of the need is created as a motive for learning; and finally, the topic is developed in a simple natural way with sufficient applications and practice to give the pupil a sense of mastery.

A Study of Vocabulary in Arithmetic

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It has long been felt that pupils did not advance in arithmetic as they should for they did not know how to interpret the problems because of an insufficient vocabulary. In order to obtain accurate data upon this contention, a detailed study was made of the vocabulary of a recent series of arithmetic textbooks. A record of the words used with the frequency of each was listed. Some of the results are noted in Table I.

The number of different words found in the testbooks for grades III, IV, V, and VI, are 1,286. From Table I we find that the frequency of use ranged from 1 to 124 in the third-grade textbook, while in the

textbook of the seventh grade the frequency ranged from 1 to 726. From this table we also find the records of the words which occurred twice, three times, four times, and five times in the textbooks of each of these four grades.

words with examples which occur in the textbook of the third, fourth, fifth, and sixth grades, but not in the textbook of any other grade. In Table III are listed all the words ranging from those having the highest frequency,

TABLE I. WORDS OCCURRING FIVE TIMES OR LESS IN THIS SERIES OF TEXTBOOKS

	Grade				
	III	IV	V	VI	All
No. of different words	502	546	792	946	1286
Range of frequency of words	1-124	1-140	1-205	1-296	1-726
No of words occurring once	164	152	222	270	315
No. of words occurring twice	73	79	101	133	155
No. of words occurring three times	35	51	53	82	79
No. of words occurring four times	27	29	36	63	61
No. of words occurring five times	10	26	38	40	48
No. of words occurring five times or less	309	337	550	588	658

In Table II the number of words occurring in the textbooks for grades IV and V which do not occur in the textbooks of the previous or succeeding grades is listed, while the words occurring in the third-grade textbook

726, to those having a frequency of 50. These are not arranged according to grade textbooks, but are listed for the entire group.

Table IV lists those words which had a frequency of less than 50,

TABLE II. NUMBER OF WORDS OCCURRING IN THE TEXTBOOK FOR A GIVEN GRADE NOT FOUND IN THOSE OF THE PREVIOUS OR SUCCEEDING GRADES AND THE NUMBER NOT FOUND IN THE TEXTBOOK OF ANY OTHER GRADE

	Number of Words	Examples
Grade III not in the grade IV	141	coin, height, absent
Grade IV not in the grade III	191	cubic, fares, million
Grade IV not in the grade V	121	collect, build, cubes
Grade V not in the grade IV	352	mills, multiple, sixths
Grade V not in the grade VI	215	ratio, coins, circle
Grade VI not in the grade V	373	percentage, volume, weighs
Grade III not in any other grade	54	select, among, copying
Grade IV not in any other grade	26	alter, intervals, margin
Grade V not in any other grade	134	stands, applied, begin
Grade VI not in any other grade	290	statement, zone, advance, angle

not found in that of the fourth grade, and those found in the textbook of the sixth grade but not found in that of the fifth grade are also listed. In addition to this we find the number of

arranged in groups from a frequency of 49 to zero.

Some of the points which have been brought out by the study are given below:

TABLE III. THE WORDS HAVING A FREQUENCY OF FIFTY OR MORE ARRANGED FROM HIGHEST FREQUENCY TO LOWEST

Word	Frequency	Word	Frequency	Word	Frequency
how many	726	sold	125	third	72
feet	655	bushels	121	tens	71
find	619	common	121	point	70
each	545	following	120	rate	70
pounds	483	part	118	cubic	69
cost	453	times	117	average	68
inches	432	year	117	gallon	68
how much	429	bushel	116	oral	67
number	424	hour	109	take	67
yards	370	per	109	measures	66
from	341	test	108	table	66
square	312	cent	106	total	66
one	285	right	106	denominator	65
numbers	266	equal	102	minute	65
pound	222	three	101	parts	65
long	220	foot	100	difference	64
days	215	read	97	month	64
cents	203	subtract	97	place	63
yard	201	fraction	97	used	63
how	194	product	94	all	62
quarts	191	length	94	draw	62
change	189	second	93	five	61
work	187	distance	90	sum	60
write	179	paid	90	quotient	60
miles	177	same	89	pint	59
two	172	pint	89	rods	59
dozen	171	gallons	88	hundredths	58
fractions	161	quart	87	addition	57
decimal	158	tons	87	between	57
divide	158	peck	85	four	57
hours	154	use	85	problems	57
bought	151	unit	83	subtraction	57
add	149	left	82	acres	56
first	149	division	80	both	56
spent	149	decimals	78	multiplication	56
called	141	solution	77	received	56
ounces	141	weeks	76	units	56
multiply	138	circle	76	rod	55
day	137	into	74	term	55
inch	133	time	73	hundred	54
make	132	years	73	ton	51
wide	132	bill	73	dollar	50
amount	131	figure	72	receive	50
price	127	more	72		
written	127	show	72		

1. It was noted that when a new word appeared it was usually repeated many times within two or three pages and then probably never used again. Examples: rod, multiply, quotient, acre, and subtrahend.

2. Many words were dropped and not used in the next higher grade: (1) words used in the third grade and in no other, quarters, quire, coin, strip, illustrate; (2) words used in the fourth grade and no other, alone, remove, cubical, credit, alter; (3) words used in the fifth grade and no other, agreed, cancel, applied, contract, accurately; and (4) words used in the sixth grade and no other, acres, analysis, percent, angle, percentage.

3. Many new words were added with each new book. *See* Table II.

TABLE IV. WORDS LISTED ACCORDING TO THEIR FREQUENCY OF USE

49–25	24–20	19–15	14–10	9–5	4–0
divided	example	mills	annex	averaged	angles
fourth	problem	percentage	circles	additional	balances
fractional	purchase	rectangle	costs	avoirdupois	coin
hundred weight	prime	seven	cube	deposit	digits
ounce	worked	sums	cancellation	eighty	drill
result	worth	solid	denominator	equivalent	debtor
six	equals	sixths	earns	edge	depth
width	expenses	weigh	factor	graph	exercise
multiplicand	degrees	zone	gross	plus	equation
count	column	numerator	perimeter	percent	invert

4. Many words were used as instruction words, that is, words directing the pupil what to do. Examples: find, add, solve, change, divide, take, measure, multiply, note, place.

5. Many words were found to have a special meaning for arithmetic. Examples: table, circle, factor.